Find $\int \frac{dx}{x^2 - 8x + 1}$

Find $\int \frac{dx}{x^2-8x+1}$

30|8|25

$$\int \frac{dx}{x^2-8x+1}$$

$$= \int \frac{dx}{(x-4)^2-15}$$

$$= \int \frac{du}{u^2-15}$$

$$= \int \frac{\sqrt{15}\sec\theta\tan\theta}{15\sec^2\theta-15}\,d\theta$$

$$= \frac{\sqrt{15}}{15}\int \frac{\sec\theta\tan\theta}{\tan^2\theta}\,d\theta$$

$$= \frac{1}{\sqrt{15}}\int \frac{1/\cos\theta}{\sin\theta/\cos\theta}\,d\theta$$

$$= \frac{1}{\sqrt{15}}\int \frac{1}{\sin\theta}\,d\theta$$

$$= -\frac{\ln|\cot\theta+\csc\theta|}{\sqrt{15}} + C$$

$$= -\frac{\sqrt{15}}{15}\ln\left|\frac{\sqrt{15}+x-4}{\sqrt{(x-4)^2-15}}\right| + C$$

$$= -\frac{\sqrt{15}}{30}\left(\ln|x-4+\sqrt{15}| - \ln|x-4-\sqrt{15}|\right) + C$$

$$x^2-8x+1$$

$$= x^2-8x+\left(\frac{8}{2}\right)^2-\left(\frac{8}{2}\right)^2+1$$

$$= (x-4)^2-15$$

$u = x-4$

and $u = \sqrt{15}\sec\theta$.

$\Rightarrow du = \sqrt{15}\sec\theta\tan\theta\,d\theta$

$$\frac{d}{d\theta}(\cot\theta+\csc\theta)$$

$$= -\csc^2\theta - \cot\theta\csc\theta$$

$$= -\csc\theta(\cot\theta+\csc\theta)$$

$$\csc\theta = -\frac{\frac{d}{d\theta}(\cot\theta+\csc\theta)}{\cot\theta+\csc\theta}$$

$$\Rightarrow \int \csc\theta\,d\theta = -\ln|\cot\theta+\csc\theta| + C$$

$$\sec\theta = \frac{x-4}{\sqrt{15}}$$